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A jump condition of PMP-based control for PHEVs

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ABSTRACT

An optimal control strategy based on Pontryagin's Minimum Principle (PMP) is a promising solution because it provides a simple solution for controlling Hybrid Electric Vehicles (HEVs) and guarantees the best performance under reasonable conditions [1,2]. However, it needs to be very careful when applying the control strategy if inequality state constraints are active because handling the state constraints is one of the difficult issues in optimal control problems. In contrast to HEVs, Plug-in Hybrid Electric Vehicles (PHEVs) possibly consume all of the available electric energy, and so the activation of the state constraint on minimum State of Charge (SOC) is, unfortunately, a very common control problem for PHEVs. This paper describes mathematical derivations for an additional condition necessary for the inequality state constraints and solves the problem with several control options. Whereas PMP-based control allows a unique solution for HEVs [2], this paper shows that the control idea based on PMP produces a number of alternative solutions for PHEVs. However, battery efficiencies can be considered to evaluate the optimality of each solution, and simulation results from several control options show that maximizing a blended-mode control is the best solution to saving fuel for PHEVs. In terms of performance, the results of applying blended-mode control are very close to those obtained by applying a global optimal solution obtained from Dynamic Programming (DP).

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1. Introduction

Energy management in Hybrid Electric Vehicles (HEVs) is a very interesting subject in that system efficiency could be improved by changing a control algorithm in the Hybrid Control Unit (HCU). This improvement could be achieved without additional costs, or sometimes, affords the opportunity to cut costs by optimizing the size of the components that have unnecessarily high capacity. For the last decade, a number of energy management strategies that aim to control the system in optimality have been vigorously researched, and several comparative studies classified the strategies according to algorithms, such as rule-based control, artificial intelligence control, and control based on optimal control theory [3–6]. One of the ideas based on optimal control theory, a control concept based on Pontryagin's Minimum Principle (PMP), was introduced as a supervisory control algorithm, and the Equivalent Consumption Minimization Strategy [7] has been interpreted in the concept of PMP [1,8,9]. On the other hand, a study has proved that PMP-based control guarantees global optimality under realistic conditions [2]. Furthermore, several researchers have shown that the control concept based on PMP is potentially promising [10–13]. As another control idea based on optimal control theory, Dynamic Programming (DP) is an outstanding method when solving the optimal control problem because it always produces an unbeatable solution, but it is not easy to directly apply the concept to real-time control. On the other hand, PMP is very easily applied to the Real World, but the optimality should be carefully verified because PMP does not automatically guarantee the optimality—it only provides necessary conditions for an optimal solution. There are several studies that proved the optimality of PMP-based control in HEVs [1,2], but the application of the control idea needs additional consideration if it is applied to Plug-in Hybrid Electrical Vehicles (PHEVs). This paper primarily describes the theoretical application when the control idea is applied to PHEV because a solution from PMP shows a different pattern if the state of the control problem is constrained by conditions of inequality. Handling inequality constraints in PMP is very complicated because the constraints possibly produce ambiguity, which is, frequently, called a jump condition [14]. In this paper, the additional jump condition for PHEVs is derived from mathematical idea, by which the condition can be explained by physical meaning because the co-state of PMP is interpreted as an equivalent weighting factor between fuel usage and electrical usage. In general, there exist two feasible control ideas for PHEVs. One idea is consuming allowable or usable electrical energy as soon as possible and sustaining State of Charge (SOC) on the depleted level. The other idea is using a blended mode as much as possible, so that the system finally consumes all of the usable electrical energy at the end of the trip. These ideas are not

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new concepts, but both of them can be realized by PMP-based control with the jump condition, and, substantially, they are optimized by the concept, so their maximum performance can be evaluated. Simulations for PHEVs based on these control ideas show that the system truly achieves optimality under the second control option in the maximized blended mode. We believe that this paper contributes modestly to an understanding of the optimal control idea for PHEVs, and so researchers will be able to use the information in this paper to create strong solutions that could be feasible energy management strategies for PHEVs.

2. Control with state constraints

Optimal control based on PMP can be considered to be a promising solution not only for HEVs but also for PHEVs. In reality, an exact solution of the control concept guarantees global optimality when the efficiency of the battery has a concave curve for SOC if there is not a state constraint [2]. It is, however, not easy to solve the control problem if the state constraints exist, such as a limitation of SOC range, and applying the control concept at the state constraints should be carefully dealt with an exact solution. In this section, necessary conditions associated with PMP with inequality state constraints are introduced to address the control issue on constrained boundaries.

2.1. Necessary conditions

As a clever interpretation of the Euler–Lagrange equation in Calculus of Variation, PMP produces necessary conditions that optimal solutions should satisfy. When the control looks for the minimum of a functional, g,

$$\min \ J = \int g(x, u, t)dt \tag{1}$$

over pairs (x, u) that satisfy $\dot{X} = f(x, u)$, the Hamiltonian of this problem is defined as:

$$H = g(x, u, t) + \lambda \cdot f(x, u) \tag{2}$$

where λ is Lagrange multiplier, which is frequently called as a co-state in optimal control problems. When the final time and the final state are fixed, the principle requires that the optimal solution satisfies the following conditions [1]:

$$\dot{x} = \frac{\partial H}{\partial \lambda} \tag{3}$$

$$\dot{\lambda} = -\frac{\partial H}{\partial x} \tag{4}$$

Further, the Hamiltonian on the optimal pair (x^*, u^*) should satisfy that

$$H(x^*, u^*, \lambda^*, t) \leq H(x^*, u, \lambda^*, t) \tag{5}$$

The input u, if there are constraints for u, should be admissible input for given x and t, and $x^*(t)$ satisfies boundary conditions at initial and final states. Simply, Eq. (3) always resulted in the state equation of $\dot{X} = f(x, u)$, and Eq. (5) is, intuitively, a quite natural condition to obtain an extreme solution—that is, the means by which the optimality cannot be achieved with u inferior to other admissible u. The only intuitively incomprehensible condition in Eq. (4), in fact, came from the state equation. In optimization problems, the multipliers are utilized for compensating the penalty of the constraint function and are determined by the relative geometries between the objective functions and the constraint functions. Because the functions generally do not depend on each other, independent multipliers are supposed to be for each constraint function. As a special case of the optimization problem, the optimal control problem could also produce an arbitrary multiplier set, λ, in each constraint f in every moment, but the multiplier can be characterized by another state equation, like Eq. (4), because the state equation, $\dot{X} = f(x, u)$, gives additional information for f according to time, which is a very unique factor that categorizes the optimal control problems under the general optimization problems. On the other hand, an important note to this optimal principle is that it does not fulfill a sufficient condition for the optimality. If a solution satisfies these necessary conditions, it is just a candidate for an absolute solution. It is, however, proved that the necessary condition can be sufficient to solve the optimal control problem of HEVs when the SOC limitations are not considered [1], and, further, the optimality is also available, even when using a wide range of SOC, if battery efficiency is a concave function of SOC [2]. In this paper, our discussions are based on the global optimality of those studies, and the optimality will be considered when the SOC limitation is applied for PHEVs.

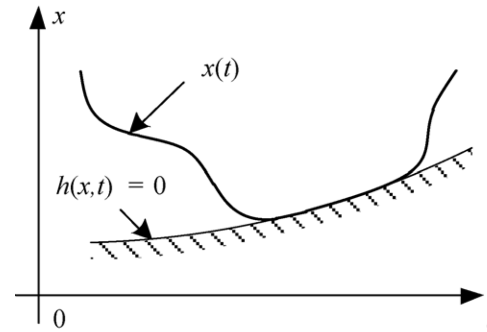

Fig. 1. An inequality state constraint, $h(x,t) \geq 0$, returns the state into the feasible area.

2.2. Inequality state constraints

As stated in the beginning of Section 2, if an inequality state constraint in Eq. (6) exists as shown in Fig. 1, the optimal control problem becomes more complicated than the problem without constraints for the state x.

$$h(x, t) \geq 0 \tag{6}$$

In this case, an optimal co-state λ^* that satisfies the necessary condition in Eq. (4) might be discontinuous—the discontinuity is not a violation of Eq. (4), but co-states without the constraint are generally continuous. For last several decades, a number of methods have been introduced to solve optimal control problems with inequality state constraints, which are, of course, very effective on various types of constrained states, but they are not causal or are, sometimes, ambiguous. Hartl et al. researched a number of methodologies and categorized them by several groups [14]. Although there are many methods, using a concept of a jump for the discontinuity of the co-state is a very useful idea to describe the solution of the inequality state constrained problems, so an additional condition that describes the jump will be briefly introduced, and the application of the control idea will be discussed in the following section. Before the discussion, it should be noted that there were two studies that solved the control problem of PHEVs by applying alternative methods. One study [11] used an additional multiplier with a direct adjoining approach [14], which is substantially based on Karush Kuhn Tucker condition, and the other one [1] utilized McGill's approach, which is based on the numerical penalty concept [15]. These are very effective approaches to find out optimal solutions, but the former solution is not easy to be designed as a forward looking controller, and the later solution includes an ambiguity because the penalty concept is not realistic. Therefore, a comprehensive approach based on jump condition is very useful for the practical control problem of PHEVs.

2.3. Jump conditions

As stated in Section 2.1, the co-state λ is characterized by the co-state equation in the optimal control problem in Eq. (4), whereas Lagrange multiplier λ is determined by the geometric relation between the objective functions and the constraint functions in general optimization problems. However, the arbitrary characteristic may appear when the inequality state constraint is active, which is called a jump condition. The mathematical condition for the idea of the jump will be derived in this section. Considering the state constraint, the functional J in Eq. (1) can be expressed as [16]:

$$J = \int_{t_0}^{t_f} \left\{ g(x, u, t) + \lambda(f - \dot{x}) \right\} dt \tag{7}$$

where t_0 and t_f are an initial time and a final time. The objective of the control is to find out an extreme trajectory that minimizes the functional in Eq. (1) or Eq. (7) while satisfying all of the boundary conditions, for which the variation of J should be zero

$$\delta J = 0 \tag{8}$$

The variation δJ can be obtained according to δx, δu, and $\delta\lambda$, which can be expressed as [17]:

$$\delta J = \lambda_0 \delta x_0 - \lambda_f \delta x_f - H_0 \delta t_0 + H_f \delta t_f + \int_{t_0}^{t_f} \left[\left\{ \dot{x} - \frac{\partial H}{\partial \lambda} \right\} \delta\lambda + \left\{ \dot{\lambda} + \frac{\partial H}{\partial x} \right\} \delta x + \frac{\partial H}{\partial u} \delta u \right] dt = 0 \tag{9}$$

where H is defined as the Hamiltonian in Eq. (2). The boundary values, x_0, λ_0, and H_0, are respectively initial values for each variable, and x_f, λ_f, and H_f are final values for them. In the general control problem, the initial time and the initial state, t_0 and x_0, are fixed because they are possibly given, so the variations for these two variables are zero, $\delta t_0 = \delta x_0 = 0$. Therefore, Eq. (9) is to be:

$$\delta J = -\lambda_f \delta x_f + H_f \delta t_f + \int_{t_0}^{t_f} \left[\left\{ \dot{x} - \frac{\partial H}{\partial \lambda} \right\} \delta\lambda + \left\{ \dot{\lambda} + \frac{\partial H}{\partial x} \right\} \delta x + \frac{\partial H}{\partial u} \delta u \right] dt = 0 \tag{10}$$

Considering that $\delta\lambda$, δx, and δu are arbitrary values, all of the terms in Eq. (10) need to be zero to satisfy $\delta J = 0$. If x_f and t_f are fixed, the first and second terms in Eq. (10) also become zero because δx_f and δt_f are zero, which allows the optimal trajectory to have any values for λ_f and H_f. Further, by setting the destinies to zero, the three terms in the integral result in the three necessary conditions in Eqs. (3)–(5). In PMP, the necessary condition requires the minimization of Hamiltonian in Eq. (5) instead of $\partial H/\partial u = 0$, which is a more generalized condition for optimality. On the other hand, we can expand the idea to a piecewise optimal control and consider δJ when x has a corner at t_j, as shown in Fig. 2.

Dividing the trajectory by two different segments, $[t_0\ t_j]$ and $[t_j\ t_f]$, the functional J can be expressed as:

$$J = \int_{t_0}^{t_j} \left\{ g(x, u, t) + \lambda(f - \dot{x}) \right\} dt + \int_{t_j}^{t_f} \left\{ g(x, u, t) + \lambda(f - \dot{x}) \right\} dt \tag{11}$$

The extreme condition in Eq. (8), $\delta J = 0$, must be satisfied though x^* has an unexpected corner. Eq. (9) can be applied to each segment

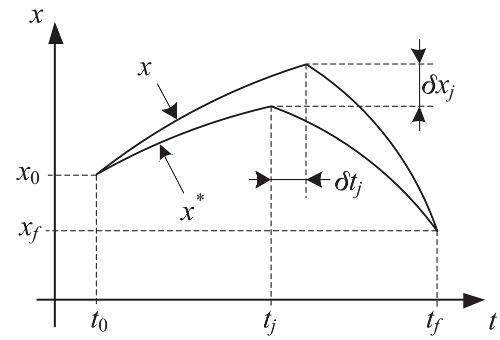

Fig. 2. A piecewise-smooth trajectory. An optimal x^* has a corner at t_j and $\dot{X}^*$ is not continuous at t_j. [17].

of the functional J when x_0, t_0, x_f, and t_f are fixed. δJ can be expressed as:

$$\delta J = -(\lambda_j^- - \lambda_j^+)\delta x_j + (H_j^- - H_j^+)\delta t_j + \int_0^{t_j} \left[\left\{ \dot{x} - \frac{\partial H}{\partial \lambda} \right\} \delta\lambda + \left\{ \dot{\lambda} + \frac{\partial H}{\partial x} \right\} \delta x + \frac{\partial H}{\partial u} \delta u \right] dt + \int_{t_j}^{t_f} \left[\left\{ \dot{x} - \frac{\partial H}{\partial \lambda} \right\} \delta\lambda + \left\{ \dot{\lambda} + \frac{\partial H}{\partial x} \right\} \delta x + \frac{\partial H}{\partial u} \delta u \right] dt = 0 \tag{12}$$

where λ_j^- and H_j^- are the values at left-hand side limit of the junction time t_j, and λ_j^+ and H_j^+ are the values at right-hand side limit. From the external terms of the integral, a condition at the corner, or at junction time t_j, is obtained, which is expressed as:

$$-(\lambda_j^- - \lambda_j^+)\delta x_j + (H_j^- - H_j^+)\delta t_j = 0 \tag{13}$$

As well as Weierstrass–Erdmann corner condition is derived for the Euler–Lagrange equation [17], a corner condition for PMP can be derived as Eq. (13). The meaning of the jump condition can be described by Eq. (13). If there does not exist any constraint barrier for x, δx_j and δt_j can be arbitrary values, so $-(\lambda_j^- - \lambda_j^+)$ and $(H_j^- - H_j^+)$ must be zero; there is no jump for the co-state or Hamiltonian when there is no barrier for the state. The jump condition, however, appears when an additional condition enforces a correlation between δx_j and δt_j (i.e., the barrier of the inequality state constrains force x to stay in a feasible area, like that shown in Fig. 3).

The virtual displacement δx_j is not an arbitrary value any more when x touches the state constraint. If x is starting to move along the inequality constraint, the constraint equation is activated, $h(x, t) = 0$,

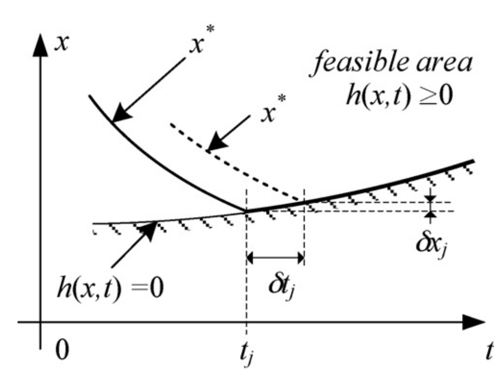

Fig. 3. The optimal trajectory x hits the state barrier at t_j, but it cannot penetrate it and possibly moves according to the barrier. On the barrier, δx_j and δt_j are not arbitrary values any more. The corner condition is also available when x just touches it and is rebounded.

and the variation of h should be zero until the state leaves the constraint, which can be expressed as:

$$\delta h = \frac{\partial h}{\partial x}\delta x + \frac{\partial h}{\partial t}\delta t = 0 \tag{14}$$

From Eqs. (13) and (14), a new necessary condition at t_j is obtained when the inequality constraint is activated:

$$\frac{\delta x_j}{\delta t_j} = \frac{H_j^- - H_j^+}{\lambda_j^- - \lambda_j^+} = -\frac{\partial h_j/\partial t}{\partial h_j/\partial x} \tag{15}$$

where δt_j is not zero. This relation is stating that

$$\lambda_j^+ = \lambda_j^- - \eta\frac{\partial h_j}{\partial x} \tag{16}$$

$$H_j^+ = H_j^- + \eta\frac{\partial h_j}{\partial t} \tag{17}$$

where η is an arbitrary value for splitting the equation. There is an additional condition that characterizes η—that is, η is always positive in our condition, but it could be different in other studies because the condition depends on the signs of the definitions [14]. It does not, however, provide important information to determine η forward in time. On the other hand, this jump can occur in every entry in which x hits the boundary, or it can occur while x is following the boundary, which is why various methodologies exist to characterize λ and η, even for an identical x. In conclusion, although Eqs. (16) and (17) do not directly give the answer for designing a controller for PHEVs, we can understand the jump condition that λ may jump to a different value when x touches the barrier of the state constraint, and this is a very useful and practical idea for solving the optimal control problem in PHEVs.

3. Optimal control for PHEVs

Beyond the mathematical derivation presented in the previous section, the goal of this study is to design a practical controller for PHEVs. In this section, the application of PMP for PHEVs will be introduced, and the energy management issue will be discussed on the basis of Charge Depleting (CD), Charge Sustaining (CS), and blended modes.

3.1. PMP on PHEVs

When State Of Charge of the battery, SOC, is the state x, and power of the battery, P_{bat}, is the control u, PMP-based control aims to find out optimal control u like shown in Eq. (5) when the Hamiltonian can be expressed as:

$$H = \dot{m}_{fc}(P_{bat}, t) + \lambda \cdot f(\mathrm{SOC}, P_{bat}) \tag{18}$$

where $\dot{m}_{fc}$ is a fuel consumption rate, and f is a state equation for SOC [1]. The control, P_{bat}, is constrained by the system's limitations, such as motor torque limit, regenerative braking limit, or battery power limit. Meanwhile, the state, SOC, is generally constrained by the limitation of SOC, i.e.,

$$\mathrm{SOC}_{\min} \le \mathrm{SOC} \le \mathrm{SOC}_{\max} \tag{19}$$

where $\mathrm{SOC}_{\min}$ and $\mathrm{SOC}_{\max}$ are minimum and maximum limitations. In the control problem for PHEVs, considering that $\mathrm{SOC}_{\min}$ is a constant, the barrier of the constraint in Fig. 3 is exactly flat. Therefore, δx_j should be zero at any junction time t_j whereas δt_j can have an arbitrary value, which means that the jump occurs only for λ, not for H [see Eq. (13)]. On the basis of the concept, a practical control solution with the jump condition can be expected as shown in Fig. 4.

When the vehicle is starting at full SOC at t_0, an initial co-state, λ_0^{cd}, can be selected for a Charge Depleting mode, and λ is calculated from the co-state equation in Eq. (4) for the depletion. If SOC hits $\mathrm{SOC}_{\min}$, λ jumps to λ_0^{cs}, which is an initial value for a Charge Sustaining mode, but the co-state equation in Eq. (4) is still available for λ. In that λ is interpreted as a weighting factor of electrical usage, the jump physically means a change in the overall pattern of electrical consumption to avoid violating the constraint. For instance, it informs the controller that the electrical energy becomes an expensive source when SOC hits the minimum limit. Now, an intricate question remains in this control concept—how we can choose λ_0^{cs} and λ_0^{cd}? In terms of the conclusion, there is no brilliant method to estimate these co-states, but alternative ways could be considered for the Real World application. For example, if λ_0^{cd} is selected as 0, the controller tries to use electricity as much as possible because the Hamiltonian would give a message that the cost of the electrical usage is very low, or zero. After consuming all possible electricity under the zero co-state, the controller can choose an appropriate λ_0^{cs} if the future driving cycle is given [10], or, if no information is given, the controller can regulate the λ^{cs} to sustain SOC [13], both of which are very practical solutions and showed good results in simulations. This control option could be a practical solution, but the problem is that the solution is unfortunately not a unique one obtained from PMP-based control. Because λ affects the pattern of SOC, selecting different λ_0^{cd} at the starting time causes a different junction time t_j, and the controller should choose another λ_0^{cs} to sustain SOC at the depleted level, at which a lot of solutions that satisfy the jump condition could exist. PMP does not state which solution is the best one because all of these solutions satisfy the necessary conditions from PMP, so the optimality will be discussed on the basis of additional information in the following section.

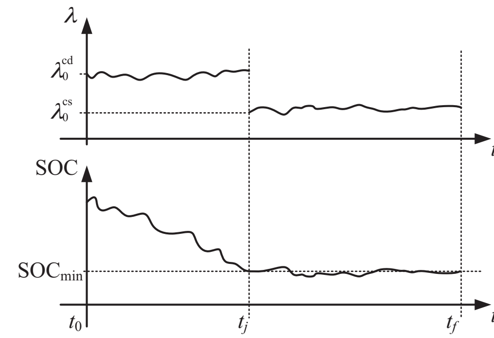

Fig. 4. Control concept for λ. While SOC is in the allowable range (>$\mathrm{SOC}_{\min}$), λ is calculated from the co-state equation according to initial values, and it jumps to another value when SOC hits the constraint.

3.2. Blended-mode control

As stated in the previous section, applying the necessary conditions to address the control problems of PHEVs does not result in a unique solution, so several strategies to realize the PMP-based control with the jump condition could be considered for the optimal solution. The easiest control is to try and not use fuel until all of the usable electrical energy is depleted, as we suggested in the previous section. In that case, the Charge Depleting (CD) mode is shortened like the $\mathrm{SOC_a}$ shown in Fig. 5 because the controller does not allow the engine to be turned on if the motors can produce all of the requested power until the usable electrical energy is exhausted. After the CD mode, the Charge Sustaining (CS) mode can be realized by an appropriate λ_0^{cs}. One of the alternatives, $\mathrm{SOC_b}$ in Fig. 5, uses a blended-mode control that allows the engine to support the propulsion power even though motors are capable of supplying all of the requested power. To realize this scenario, a well-estimated initial λ_0 at t_0 is absolutely necessary, so that SOC does not touch the limit, and the system consumes all electrical energy at t_f, as

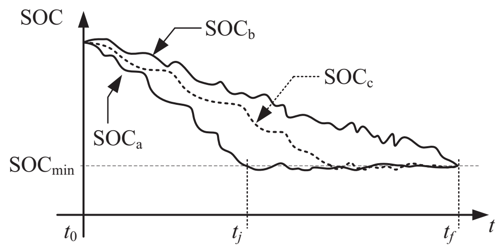

Fig. 5. SOC_a tries to use pure-electric mode as much as possible, whereas SOC_b utilizes the blended mode until the end of the driving cycle.

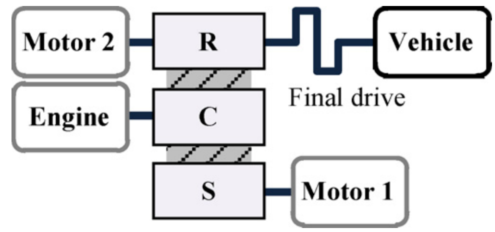

Fig. 6. The simplified configuration of the power split hybrid system used in this study.

shown in Fig. 5. Both of SOC_a and SOC_b should be considered as optimal solutions that satisfy the necessary conditions of PMP. The difference between the two is that SOC_a requires a jump when the system turns from CD mode to CS mode, but SOC_b does not have the discontinuous λ because the constraint has not been activated. Because both of these control options fulfilled the optimality, the performance of the control cannot be compared by PMP. Instead, it needs to consider the battery efficiency to compare these two control scenarios.

If the battery were very efficient at low SOC (like SOC_{min}), SOC_a would be superior to SOC_b because SOC_a, intuitively, operate in the efficient range more so than SOC_b. In the real world, however, SOC_a does not operate in the efficient range because battery is generally inefficient near SOC_{min}—that is one reason why there is a limitation of minimum SOC. Under a realistic battery model, a previous study showed that the SOC_b is a global and unique solution when there is no limitation for SOC [2], which is the case where the vehicle is controlled by a continuous λ without jump. According to the proof, there is no alternative solution that can defeat the continuous λ if the efficiency curve is concave for SOC, and if the efficiency near SOC_{min} is lower than other SOC. By expanding this idea, a conclusion can be obtained that PMP-based control always achieves the better performance if the blended mode lasts for as long as possible: $SOC_b > SOC_c > SOC_a$. In that the blended mode has more opportunities to control the power management than all-electric mode, this conclusion is reasonable. However, choosing an appropriate initial λ_0 at t_0 for the all-blended mode is not convenient when the driving schedule is not given prior, and so a practical concept for the blended mode could be truly realized with intelligent devices based on a Global Positioning System, or it could be applied to vehicles driven under predictable patterns, like buses on regular routes [12].

4. Optimal control simulation

Three control scenarios are used in this study. The first is the control that maximizes the blended mode, which produces a trajectory of SOC like SOC_b in Fig. 5. The second is the control that uses the all-electric mode—the engine produces power only when the motors does not supply all the requested power—and CS mode, like SOC_a, in which the co-state λ needs to jump when entering CS mode. The third is a solution from Dynamic Programming that produces an absolute trajectory, which is used to verify the optimal solution from PMP.

4.1. Simulation and vehicle model

The vehicle model used in this study is a power split hybrid system that has a single planetary gear set as a power split device, like that shown in Fig. 6. All of the data for the component models and vehicle model are based on a 2004 Toyota Prius in Autonomie developed by Argonne National Laboratory (see Table 1) [18]. Especially, the battery capacity is expanded from 1.3 kW h to 5.2 kW h to realize the PHEV's performance [19].

To solve the optimal control problem, OC_SIM, which was developed by Seoul National University, is used for each control option. The simulator is designed for users to easily define the vehicle's configuration and to select the components from its database. The simulator is able to produce the optimal control trajectory solved from both PMP and DP by using a backward-looking simulation. The vehicle model is evaluated on the Urban Dynamometer Driving Schedule (UDDS) extended to 5 times the cycle, which is selected to have sufficient distance to observe the differences among the control options when different initial co-states are chosen. This study is only focusing on a relative comparison of the optimal control concepts, whereas selecting a representative cycle or sizing a battery capacity could be another critical issue for evaluating the performance of PHEVs [19].

4.2. Simulation results

As the condition of PHEVs, the simulation used the initial SOC as 90% and allowed the system to consume the electrical energy until SOC fell to 20%. By selecting multiple initial co-states, different solutions can be obtained from PMP control. Fig. 7 shows three different SOC trajectories solved by PMP and one SOC trajectory solved by DP.

First, the trajectory 'PMP jump 1' is the solution when λ_0^{cd} is zero at starting time. In that case, the system tries to consume the electrical energy as much as possible until SOC hits the constraint, and, when SOC does hit the constraint, the co-state jumps to an appropriate λ_0^{cs}, so SOC is sustained until the time horizon, which is the case of SOC_a described in section 3.2. 'PMP jump 2,' is another case of PMP solution in which the initial co-state λ_0^{cd} is smaller than the value of 'PMP jump 1,' which is able to extend CD mode up to about 5000 s by using the blended-mode control, but it still needs the jump of the co-state when entering to CS mode. On the other hand, the SOC trajectory 'PMP exact' is obtained by selecting an appropriate λ_0, so that the system uses only the blended mode for the entire driving cycle. To find out the exact λ_0 from iterative simulations, the simulator uses multiple shooting methods combined with a variation of extremals [1]. The trajectory 'DP' is introduced to prove that 'PMP exact' is the best real solution of the three PMP solutions.

Table 1
Vehicle parameters used in simulations.

Vehicle total mass	1490 kg
Engine	1500 cc (peak power: 43 kW)
Motor1	25 kW (peak power: 50 kW)
Motor2	15 kW (peak power: 30 kW)
Battery	5.2 kW h
Planetary gear ratio	2.6 (78/30)
Final gear ratio	4.113
Rolling resistance coefficient	0.007 + 0.00012 × vehicle velocity
Frontal area	2.25 m^2
Drag coefficient	0.29
Wheel radius	0.305 m
Air density	1.23 kg m^{-3}

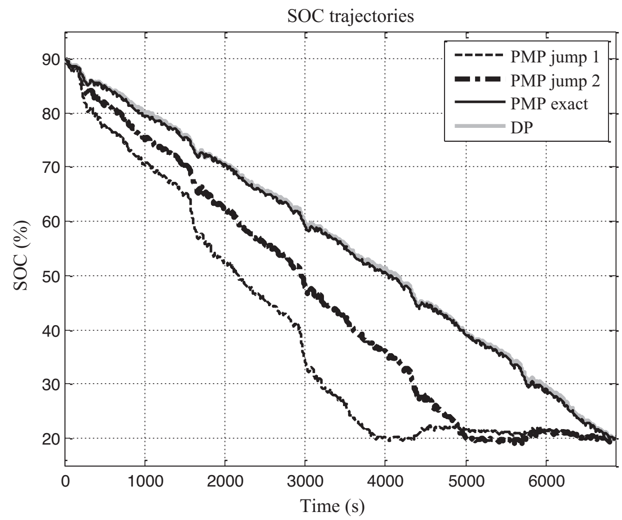

Fig. 7. SOC trajectories when jump condition is applied. 'PMP exact' indicates the optimal solution when there is no jump, and DP solution shows that 'PMP exact' is the optimal solution.

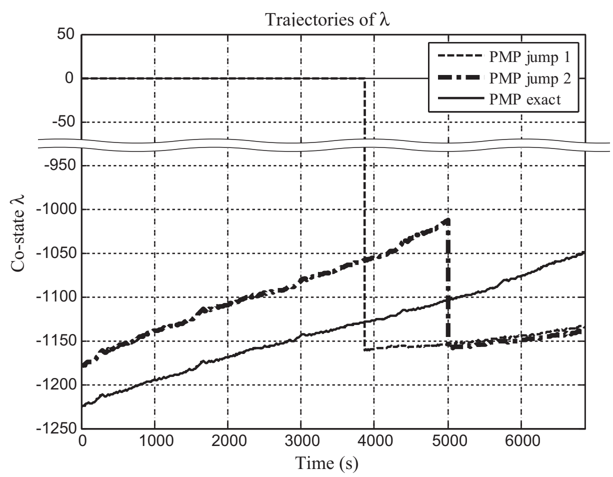

Fig. 8. Trajectories of co-states are discontinuous when SOC hits the constraint.

Optimal trajectories of co-states for the three solutions are shown in Fig. 8, in which the co-states for 'PMP jump 1' and 'PMP jump 2' jump to other co-states when SOC hits 20%, and λ_0^{cd} and λ_0^{cs} for the trajectories are shown in Table 2. On the other hand, the co-state could be considered to be a near constant value for the control problem of HEVs [1,9], and the co-states shown for CS modes in Fig. 8 are not changed much, which makes the controller simple when it is applied to the real vehicle. However, the co-states in CD modes show relatively greater change than the CS modes, so it is not a good approach to use a constant co-state for the control of PHEVs.

The objective of the control is to minimize fuel consumption, and so the performance of the control is evaluated on the basis of total fuel consumption, as shown in Fig. 9.

Table 2
Jump conditions of each solution.

	Initial λ or λ_0^{cd}	λ_0^{cs}	Entry time (s)
PMP jump 1	0	−1160.53	3874
PMP jump 2	−1180	−1158.87	5011
PMP exact	−1225.72	–	–

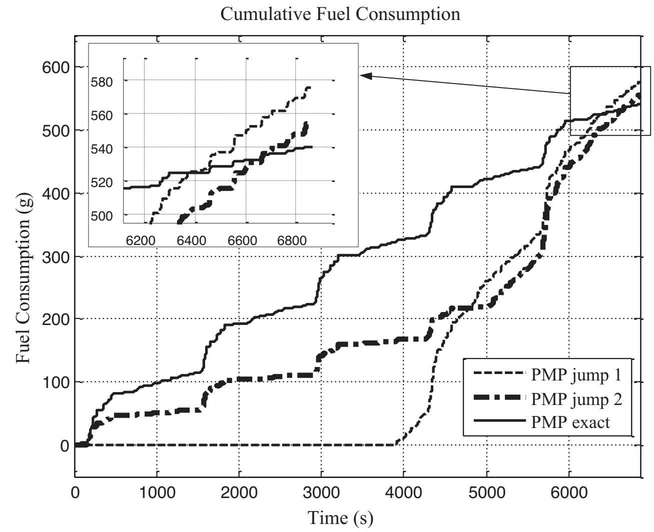

Fig. 9. The results of total fuel consumption according to jump conditions.

As we discussed in Section 3.2, the solution from 'PMP exact' that maximizes the blended mode achieves the best performance of the three PMP solutions. As long as the battery has higher efficiency at middle range of SOC than at low range, it is quite natural that the optimal solution would not stay at low SOC. To compare the exact performance of the control options, corrected fuel consumptions are calculated. By selecting appropriate initial and jumped co-states, the final SOC of each solution is very close to the desired final value. However, it is not possible to numerically select a co-state that exactly produces a desired final SOC, and so the corrected fuel consumption is used to compare the performances of the control options.

For example, the fuel consumption of 'PMP exact' is 539.55 g, while the final SOC is 19.94%, which is not the desired value of 20%. The fuel consumption is shifted to equivalent fuel consumption at 20% according to the optimal pattern of fuel consumption obtained from DP, as shown in Fig. 10, by which the performance can be evaluated at the equivalent usage of electricity. The results for the three PMP solutions are shown in Table 3 with the corrected values.

If the controller realizes a blended mode for the entire driving cycle, it can save about 6% in fuel consumption compared to using

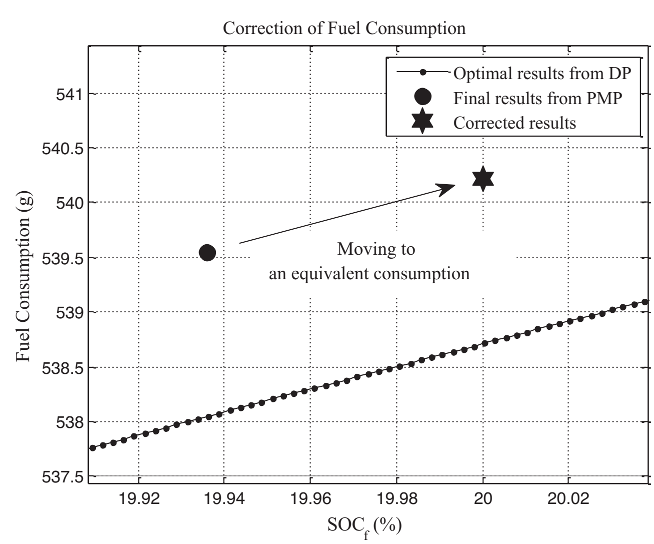

Fig. 10. An example of the correction for exact PMP solution.

Table 3
Total fuel consumption of each control.

	Fuel consumption (g)	SOC_f (%)	Corrected fuel consumption (g)	Fuel save (%)
PMP jump 1	575.134	20.0005	575.128	–
PMP jump 2	553.858	19.9726	554.141	+3.65
PMP exact	539.550	19.9359	540.214	+6.07
DP	538.710	20.0000	–	+6.33

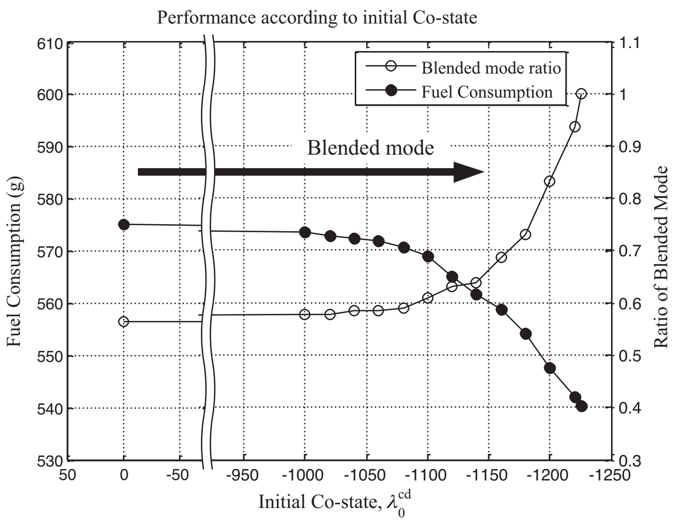

Fig. 11. The fuel consumption and the ratio of the blended mode to the entire driving cycle, according to co-states.

the all-electric driving mode. The fuel savings and the ratio of the blended mode according to the co-state are shown in Fig. 11.

In the figure, the ratio of the blended mode is defined as:

$$\text{Ratio of blended mode} = \frac{\text{Driving time of a blended mode}}{\text{Driving time of a entire trip}} \tag{20}$$

Although the co-state of 'PMP jump 1' starts with zero for λ_0^{cd}, we could obtain very similar SOC trajectories if λ_0^{cd} is -1000 because the Hamiltonian mostly produces a control signal that enforces to use the electrical energy as much as possible in this range of the co-state from 0 to -1000. The blended mode is effectively extended when λ_0^{cd} is in the range from -1000 to -1225. If the initial co-state is smaller than the optimal co-state (-1225.72), the control based on PMP cannot produce an optimal solution that satisfies the boundary condition, $SOC_f = 20\%$.

5. Conclusion

Pontryagin's Minimum Principle requires an additional condition when solving an optimal control problem with inequality state constraints. A jump condition can be utilized to satisfy the state constraints, which makes the co-state discontinuous when the constraint is active. The behavior of the co-state under the jump condition is complicated because not only the concept of jump needs to be carefully derived by mathematical consideration based on Calculus of Variation, but also the exact amount of the jump is not determined forward in time. Optimal control for PHEVs has the same issue as the stated control problem because PHEVs possibly consume all usable electric energy, and a controller constrains the SOC of PHEVs, so that it does not operate lower than a certain value, such as $SOC \geq SOC_{min}$. In this study, the control problem of PHEVs is described by focusing on the jump condition, and it shows that the jumped co-state physically means the change of the usage pattern of electrical energy for not violating the constraints. Several control options that satisfy the necessary conditions of PMP are introduced according to the jump condition. On the basis of a backward simulation technique, maximizing the blended mode is the best control solution to minimize fuel consumption—optimal control maximizes the blended mode and results in fuel savings of up to 6% on an about 35 miles urban cycle, compared to the control option that consumes the available electricity as soon as possible. Although the PMP-based control with the jump condition very well explains the behavior of the optimal control for PHEVs, applying the control concept to the Real World is another problem because lots of experience is essential to design a controller that approximates a good co-state. However, understanding the behavior of the co-state on the control problem is an important step in applying the control concept to the Real World.

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